

Topic 6 · Sheet A: Questions

Counting: permutations, combinations and arrangements

Suggested time

About 75 minutes

Total marks

73

Questions

12

Builds on

Topics 1 to 5

How to use this sheet. Work on your own paper. The questions start gently and get harder; questions 11 and 12 are exam style. Marks are a guide to how much working is expected, roughly one minute per mark. You need the multiplication rule of counting, permutations $n!$ and $\frac{n!}{(n-k)!}$, binomial coefficients $\binom{n}{k}$, arrangements of objects with repeats, and unordered samples with replacement, $\binom{n+k-1}{k}$ (stars and bars).

Tip. Before counting, decide whether order matters and whether repeats are allowed. Leave large answers in terms of $\binom{n}{k}$ and factorials unless asked for a number.

1

4 marks

- (a) In how many orders can 5 different books stand on a shelf? [1]
- (b) In how many ways can a gold, silver and bronze medal go to 3 of 7 runners? [1]
- (c) In how many ways can 3 of 7 runners be chosen for a team? [1]
- (d) How many 4-digit PINs are there, if digits may repeat? [1]

2

4 marks

- (a) How many outfits can be made from 3 shirts, 4 pairs of trousers and 2 pairs of shoes? [1]
- (b) How many number plates have 2 letters followed by 3 digits? [1]
- (c) How many 4-digit PINs have all their digits different? [1]
- (d) How many strings of 8 binary digits are there? [1]

3

6 marks

- (a) How many different arrangements are there of the letters of BANANA? [2]
- (b) How many different arrangements are there of the letters of STATISTICS? [2]
- (c) An arrangement of BANANA is chosen at random. Find the probability that it starts with B. [2]

4

6 marks

A committee of 4 is chosen at random from 6 men and 5 women.

- (a) How many committees are possible? [1]
- (b) How many have exactly 2 women? [2]
- (c) How many have at least one woman? [2]
- (d) Find the probability that the committee has exactly 2 women. [1]

5

6 marks

A hand of 5 cards is dealt from a shuffled deck of 52.

- (a) How many hands are there? [1]
- (b) Find the probability that all five cards are hearts. [2]
- (c) Find the probability of exactly two aces. [3]

6

6 marks

- (a) In how many ways can 6 identical sweets be shared among 3 children, if a child may get none? [2]
- (b) In how many ways, if every child gets at least one? [2]
- (c) How many solutions in non-negative integers does $x_1 + x_2 + x_3 + x_4 = 10$ have? [2]

7

6 marks

A sample of size 3 is taken from a population of 5 individuals. How many samples are there if they are:

- (a) ordered, with replacement; [1]
- (b) ordered, without replacement; [1]
- (c) unordered, without replacement; [2]
- (d) unordered, with replacement? [2]

8

6 marks

A lottery draws 6 different numbers from $1, \dots, 49$. You choose 6 numbers.

- (a) How many possible draws are there? [1]
- (b) Find the probability that you match all six. [1]
- (c) Find the probability that you match exactly three. [2]
- (d) Find the probability that you match none. [2]

9

6 marks

- (a) Explain, by counting, why $\binom{10}{3} = \binom{10}{7}$. [2]
- (b) Check that $\binom{8}{3} = \binom{7}{2} + \binom{7}{3}$, and explain it by whether a particular object is chosen. [2]
- (c) Explain why $\sum_{k=0}^5 \binom{5}{k} = 32$. [2]

10

7 marks

A password has 6 characters, each a lowercase letter (26 choices) or a digit (10 choices).

- (a) How many passwords are there? [1]
- (b) How many contain at least one digit? [2]
- (c) How many contain exactly two digits? [2]
- (d) A password is chosen at random. Find the probability that it contains exactly two digits. [2]

11

8 marks

Exam style. A hand of 5 cards is dealt from a shuffled deck of 52.

- (a) A full house is three cards of one rank and two of another. Find the probability of a full house. [4]
- (b) Two pairs is two cards of one rank, two of a second rank, and one card of a third rank. Find the probability of two pairs. [4]

12

8 marks

Exam style. Six dice are rolled.

- (a) Find the probability that all six show different numbers. [2]
- (b) Find the probability of exactly two sixes. [2]
- (c) Find the probability that exactly three different numbers appear, each exactly twice. [3]
- (d) If the order is ignored, how many different outcomes are there? [1]

End of questions. Worked answers and the mark scheme are in the matching answers PDF.