

Topic 5 · Sheet A: Questions

Sample spaces, events and equally likely outcomes

Suggested time

About 75 minutes

Total marks

73

Questions

12

Builds on

Topics 1 to 4

How to use this sheet. Work on your own paper. The questions start gently and get harder; questions 11 and 12 are exam style. Marks are a guide to how much working is expected, roughly one minute per mark. You need sample spaces (including Cartesian products), events as subsets, the set operations and De Morgan's laws, and $\mathbb{P}(A) = \frac{|A|}{|\Omega|}$ when Ω is finite and its outcomes are equally likely.

Notation. $A^c = \Omega \setminus A$. $\{1, \dots, 6\}^2$ is the set of ordered pairs of die results. Dice and coins are fair.

1

4 marks

Write down a sample space for each experiment, and the number of outcomes.

- (a) A coin is tossed and then a die is rolled. [1]
- (b) Three coins are tossed one after another. [1]
- (c) Two cards are drawn in order, without replacement, from cards numbered 1, 2, 3, 4. [1]
- (d) Two of the numbers 1, 2, 3, 4 are chosen at once, so their order is not recorded. [1]

2

4 marks

Two dice are rolled, with $\Omega = \{1, \dots, 6\}^2$. Let A be "the sum is even" and B be "the first die shows 1".

- (a) Find $|A|$. [1]
- (b) List the outcomes in $A \cap B$. [1]
- (c) Find $|A \cup B|$. [1]
- (d) Describe $A^c \cap B^c$ in words, and find how many outcomes it has. [1]

3

6 marks

A die is rolled, with $\Omega = \{1, \dots, 6\}$. Let $A = \{1, 2, 3\}$ and $B = \{2, 4, 6\}$.

- (a) Find $(A \cup B)^c$ and $A^c \cap B^c$, and check that they are equal. [2]
- (b) Find $(A \cap B)^c$ and $A^c \cup B^c$, and check that they are equal. [2]

- (c) Write “exactly one of A and B happens” using set operations, and list its outcomes. [2]

4

6 marks

A coin is tossed until the first head appears.

- (a) Describe a suitable sample space. Remember the possibility that no head ever appears. [2]
- (b) Write down the event E_n that the first head is on toss n . [1]
- (c) Write the event “an even number of tosses is needed” using the events E_n . [1]
- (d) Find the probability of the event in (c). [2]

5

6 marks

The numbers 1, 2, 3, 4, 5 are arranged in a random order, every order being equally likely.

- (a) How many outcomes are there? [1]
- (b) Find the probability that 1 comes first. [1]
- (c) Find the probability that 1 and 2 are next to each other. [2]
- (d) Find the probability that 1 comes before 2. [2]

6

6 marks

A bag holds one red, one green and one blue ball.

- (a) Two balls are drawn in order without replacement. Write down the sample space. [1]
- (b) Write down the sample space if the order is not recorded. [1]
- (c) Find the probability that the red and green balls are drawn, using each sample space. [2]
- (d) Now two balls are drawn with replacement and the order is not recorded. Explain why the outcomes are not equally likely, giving the probabilities of $\{R, R\}$ and $\{R, G\}$. [2]

7

6 marks

Two dice are rolled. Let A be “the first die shows at least 5” and B be “the sum is at most 4”.

- (a) Show that A and B cannot both happen. [2]
- (b) Find $\mathbb{P}(A \cup B)$. [2]
- (c) Find $\mathbb{P}(A^c \cap B^c)$. [2]

8

6 marks

An integer is chosen at random from $1, 2, \dots, 60$. Find the probability that it is divisible by:

- (a) 4; [1]
- (b) 6; [1]
- (c) both 4 and 6; [2]
- (d) 4 or 6. [2]

9

6 marks

- (a) Two dice are rolled and only the sum is recorded, so $\Omega = \{2, 3, \dots, 12\}$. Explain why $\mathbb{P}(\text{sum} = 2) \neq \frac{1}{11}$. [2]
- (b) Three coins are tossed and only the number of heads is recorded. Give the probability of each outcome. [2]
- (c) Explain why “choose a positive integer, all equally likely” cannot be modelled with $\mathbb{P}(A) = \frac{|A|}{|\Omega|}$. [2]

10

7 marks

A coin is tossed, a die is rolled, and a card is drawn whose colour (red or black) is recorded, each fairly.

- (a) Write down Ω and find $|\Omega|$. [1]
- (b) Find the probability of “head, even number and red card”. [2]
- (c) Find the probability of “head or a six”. [2]
- (d) Describe the complement of “head or a six” in words, and find its probability. [2]

11

8 marks

Exam style. A die is rolled three times, with $\Omega = \{1, \dots, 6\}^3$.

- (a) Find $|\Omega|$. [1]
- (b) Find the probability that all three results are equal. [1]
- (c) Find the probability that the results are strictly increasing. [2]
- (d) Find the probability that the results include at least one 1 and at least one 2. [2]
- (e) If the order is forgotten, compare the probabilities of seeing $\{1, 1, 2\}$ and $\{1, 2, 3\}$. [2]

12

8 marks

Exam style. A coin is tossed. If it shows heads, a die is rolled; if tails, the coin is tossed again.

- (a) Write down a sample space with 8 outcomes. [2]
- (b) Explain why its outcomes are not equally likely, and give the probability of each. [2]
- (c) Find the probability that a head appears at some point. [2]
- (d) Find the probability that a die is rolled and shows an even number. [2]

End of questions. Worked answers and the mark scheme are in the matching answers PDF.