

Topic 5 · Sheet A: Worked answers and mark scheme

Sample spaces, events and equally likely outcomes

Suggested time

About 75 minutes

Total marks

73

Questions

12

Builds on

Topics 1 to 4

How marks are awarded. Each mark has a code, as on most UK exam papers.

B A mark for a correct statement or value on its own. It does not depend on any other mark.

M A method mark, for a correct approach, even if the final answer is wrong.

A An accuracy mark, for the correct answer *after* a correct method. It cannot be awarded without the M mark before it.

E An explanation mark, for a clear reason in words. A correct reason that is worded differently scores in full.

For true or false questions, the mark is for the correct decision; where a reason is asked for, it must be the right reason.

1 Sample spaces

4 marks

- (a) $\{H, T\} \times \{1, \dots, 6\}$, 12 outcomes. (b) $\{H, T\}^3$, 8. (c) **ordered pairs (a, b) with $a \neq b$** , 12. (d) **2-element subsets of $\{1, 2, 3, 4\}$** , 6.

B1 each part correct

4

Why this question Ordered samples use tuples; unordered ones use sets. (c) has twice as many outcomes as (d), because each pair can come in two orders.

2 Events as sets

4 marks

- (a) **18**. (b) $(1, 1), (1, 3), (1, 5)$. (c) $18 + 6 - 3 = \mathbf{21}$. (d) **The sum is odd and the first die is not 1**: $18 - 3 = \mathbf{15}$ outcomes.

B1 each part correct

4

Common mistake In (c), adding $18 + 6$. The three outcomes in $A \cap B$ would be counted twice.

3 De Morgan's laws

6 marks

- (a) $A \cup B = \{1, 2, 3, 4, 6\}$, so $(A \cup B)^c = \{5\}$. $A^c \cap B^c = \{4, 5, 6\} \cap \{1, 3, 5\} = \{5\}$. **Equal.**
- (b) $A \cap B = \{2\}$, so $(A \cap B)^c = \{1, 3, 4, 5, 6\}$. $A^c \cup B^c = \{4, 5, 6\} \cup \{1, 3, 5\} = \{1, 3, 4, 5, 6\}$. **Equal.**
- (c) $(A \cap B^c) \cup (A^c \cap B) = \{1, 3\} \cup \{4, 6\} = \{1, 3, 4, 6\}$.

M1	(a)	1
A1	(a)	1
M1	(b)	1
A1	(b)	1
B1	(c) expression	1
B1	(c) outcomes	1

Why this question "Not (A or B)" means "not A and not B ". De Morgan's laws are this everyday logic written in sets.

4 Waiting for a head

6 marks

- (a) $\Omega = \{H, TH, TTH, TTTH, \dots\} \cup \{TTT \dots\}$: one outcome for each possible first head, plus the infinite sequence of tails.
- (b) $E_n = \{T^{n-1}H\}$, the single outcome of $n - 1$ tails then a head.
- (c) $E_2 \cup E_4 \cup E_6 \cup \dots = \bigcup_{k \geq 1} E_{2k}$.
- (d) The E_{2k} are disjoint, so $\sum_{k \geq 1} \left(\frac{1}{2}\right)^{2k} = \frac{1/4}{1 - 1/4} = \frac{1}{3}$.

B1	(a)	1
B1	(a) the never-a-head outcome	1
B1	(b)	1
B1	(c)	1
M1	(d)	1
A1	(d)	1

Why this question The infinite sequence of tails has probability 0, but it is still a possible outcome and belongs in Ω .

5 Random orders

6 marks

- (a) $5! = 120$.
- (b) $\frac{1}{5}$.
- (c) Glue 1, 2 into a block: $4!$ arrangements, times 2 orders inside the block. $\frac{48}{120} = \frac{2}{5}$.
- (d) Swapping 1 and 2 matches each order with 1 first to one with 2 first: $\frac{1}{2}$.

B1	(a)	1
B1	(b)	1
M1	(c) block method	1
A1	(c)	1
M1	(d) symmetry	1
A1	(d)	1

Common mistake In (c), forgetting that the block can be 12 or 21. That gives $\frac{1}{5}$, half the right answer.

6 Ordered or not

6 marks

- (a) $(R, G), (R, B), (G, R), (G, B), (B, R), (B, G)$.
- (b) $\{R, G\}, \{R, B\}, \{G, B\}$.
- (c) Ordered: (R, G) and (G, R) , so $\frac{2}{6} = \frac{1}{3}$. Unordered: $\{R, G\}$, so $\frac{1}{3}$.
- (d) $\{R, R\}$ comes only from (R, R) : $\frac{1}{9}$. $\{R, G\}$ comes from (R, G) and (G, R) : $\frac{2}{9}$.

B1	(a)	1
B1	(b)	1
B1	(c) ordered	1
B1	(c) unordered	1
E1	(d)	1
B1	(d) both probabilities	1

Why this question Forgetting the order is harmless without replacement, but with replacement it merges different numbers of ordered outcomes.

7 Disjoint events

6 marks

- (a) If the first die shows at least 5, the sum is at least 6, so it cannot be at most 4. $A \cap B = \emptyset$.
- (b) $|A| = 12$ and $|B| = 1 + 2 + 3 = 6$, so $\frac{18}{36} = \frac{1}{2}$.

(c) By De Morgan, $A^c \cap B^c = (A \cup B)^c$: $\frac{1}{2}$.

E2	(a)	2
M1	(b)	1
A1	(b)	1
M1	(c) De Morgan	1
A1	(c)	1

Why this question For disjoint events the probability of the union is simply the sum.

8 Divisibility

6 marks

(a) $\frac{15}{60} = \frac{1}{4}$. (b) $\frac{10}{60} = \frac{1}{6}$. (c) Divisible by both exactly when divisible by 12: $\frac{5}{60} = \frac{1}{12}$. (d) $\frac{15+10-5}{60} = \frac{1}{3}$.

B1	(a)	1
B1	(b)	1
M1	(c) lcm is 12	1
A1	(c)	1
M1	(d)	1
A1	(d)	1

Common mistake In (c), using $4 \cdot 6 = 24$. A number divisible by both 4 and 6 is divisible by their lowest common multiple, 12.

9 When outcomes are not equally likely

6 marks

(a) A sum of 2 needs (1, 1), while a sum of 7 has six ordered pairs. So the sums are not equally likely: in fact $\mathbb{P}(\text{sum} = 2) = \frac{1}{36}$.

(b) $0: \frac{1}{8}, 1: \frac{3}{8}, 2: \frac{3}{8}, 3: \frac{1}{8}$.

(c) Ω is infinite, so $|\Omega|$ is not a number to divide by. And if every integer had the same probability c , the total would be 0 if $c = 0$ and infinite if $c > 0$, never 1.

E1	(a)	1
B1	(a)	1
B1	(b)	1
B1	(b)	1
E2	(c)	2

Why this question Choosing the right sample space, one whose outcomes really are equally likely, is often the whole difficulty.

10 A three-part experiment

7 marks

- (a) $\{H, T\} \times \{1, \dots, 6\} \times \{\text{red, black}\}$, with 24 outcomes.
 (b) $1 \cdot 3 \cdot 1 = 3$ outcomes: $\frac{1}{8}$.
 (c) Heads: 12 outcomes; six: 4; both: 2. $\frac{12+4-2}{24} = \frac{7}{12}$.
 (d) **Tails and not a six**: $1 \cdot 5 \cdot 2 = 10$ outcomes, $\frac{5}{12}$.

B1	(a)	1
M1	(b)	1
A1	(b)	1
M1	(c)	1
A1	(c)	1
B1	(d) description	1
B1	(d) probability	1

Why this question The complement in (d) is a product of simple pieces, so it is easier to count than “head or six” itself.

11 Exam style: three rolls

8 marks

- (a) **216**.
 (b) $\frac{6}{216} = \frac{1}{36}$.
 (c) Each set of three different values gives one increasing order: $\frac{\binom{6}{3}}{216} = \frac{20}{216} = \frac{5}{54}$.
 (d) The complement is “no 1 or no 2”: $5^3 + 5^3 - 4^3 = 186$ outcomes. So $\frac{216-186}{216} = \frac{5}{36}$.
 (e) $\{1, 1, 2\}$ comes from 3 ordered outcomes and $\{1, 2, 3\}$ from 6: $\frac{3}{216}$ **against** $\frac{6}{216}$.

B1	(a)	1
B1	(b)	1
M1	(c)	1
A1	(c)	1
M1	(d) complement and inclusion–exclusion	1
A1	(d)	1
B1	(e)	1
B1	(e)	1

Common mistake In (d), counting $1 \cdot 1 \cdot 6$ for “a 1, a 2, then anything”. That ignores the order and double counts. The complement is far safer.

12 Exam style: a two-stage experiment

8 marks

- (a) $\{(H, 1), \dots, (H, 6), (T, H), (T, T)\}$.
- (b) Each (H, k) has probability $\frac{1}{2} \cdot \frac{1}{6} = \frac{1}{12}$, and $(T, H), (T, T)$ each have $\frac{1}{4}$.
- (c) All six (H, k) and (T, H) : $\frac{1}{2} + \frac{1}{4} = \frac{3}{4}$.
- (d) $(H, 2), (H, 4), (H, 6)$: $\frac{1}{4}$.

B1	(a)	1
B1	(a)	1
E1	(b)	1
B1	(b)	1
M1	(c)	1
A1	(c)	1
M1	(d)	1
A1	(d)	1

Why this question A sample space can be perfectly correct and still have outcomes with different probabilities. Then $\frac{|A|}{|\Omega|}$ is the wrong tool.

Total: 73 marks. Spotted a mistake? Report it at unimixedtutor.com and we will fix it.