

Topic 4 · Sheet A: Questions

Independence

Suggested time

About 75 minutes

Total marks

73

Questions

12

Builds on

Topics 1 to 3

How to use this sheet. Work on your own paper. The questions start gently and get harder; questions 11 and 12 are exam style. Marks are a guide to how much working is expected, roughly one minute per mark. You need the definition of independence, $\mathbb{P}(A \cap B) = \mathbb{P}(A)\mathbb{P}(B)$, its link with conditional probability, and the difference between pairwise independence (every pair) and mutual independence (every finite sub-collection).

Notation. Dice are fair and six-sided; separate rolls and tosses are independent. Give exact answers where you can, or decimals to 3 significant figures.

1

4 marks

A die is rolled. Let $A = \{2, 4, 6\}$, $B = \{1, 2\}$ and $C = \{1, 2, 3\}$.

- (a) Are A and B independent? [1]
- (b) Are A and C independent? [1]
- (c) Are B and C independent? [1]
- (d) Can two events that cannot happen together, each with positive probability, be independent? [1]

2

4 marks

A and B are independent, with $\mathbb{P}(A) = 0.3$ and $\mathbb{P}(B) = 0.6$. Find:

- (a) $\mathbb{P}(A \cap B)$; [1]
- (b) $\mathbb{P}(A \cup B)$; [1]
- (c) $\mathbb{P}(A \cap B^c)$; [1]
- (d) the probability that neither happens. [1]

3

6 marks

Let A and B be independent.

- (a) Show that A and B^c are independent. [3]
- (b) Deduce that A^c and B^c are independent. [2]
- (c) Hence write $\mathbb{P}(A \cup B)$ in terms of $\mathbb{P}(A)$ and $\mathbb{P}(B)$ using complements. [1]

4

6 marks

A machine has independent parts A , B and C , which work with probabilities 0.9, 0.8 and 0.95.

- (a) A and B are in series (both must work). Find the probability that this pair works. [2]
- (b) A and B are in parallel (at least one must work). Find the probability that this pair works. [2]
- (c) The parallel pair from (b) is in series with C . Find the probability that the whole machine works. [2]

5

6 marks

- (a) Find the probability of at least one 6 in 4 rolls of a die. [2]
- (b) Find the probability of at least one double six in 24 rolls of a pair of dice. [2]
- (c) Which of (a) and (b) is more likely? [1]
- (d) What is the smallest number of rolls of a die that makes at least one 6 more than 90% likely? [1]

6

6 marks

- (a) Of 100 students, 40 take Statistics, 30 take Physics and 12 take both. For a student chosen at random, are “takes Statistics” and “takes Physics” independent? [2]
- (b) If instead 15 took both, would they be independent? Compare $\mathbb{P}(\text{Statistics} \mid \text{Physics})$ with $\mathbb{P}(\text{Statistics})$. [2]
- (c) If 50 take Statistics and 30 take Physics, how many must take both for the events to be independent? [2]

7

6 marks

- (a) How many equations must hold for three events to be mutually independent? [1]
- (b) How many equations must hold for n events to be mutually independent? [2]
- (c) A coin is tossed three times and A_i is “head on toss i ”. Show that A_1, A_2, A_3 are mutually independent. [2]
- (d) How many equations are needed for four events? [1]

8

6 marks

- (a) $\mathbb{P}(A) = 0.4$, $\mathbb{P}(B) = 0.5$ and $\mathbb{P}(A \mid B) = 0.4$. Are A and B independent? Find $\mathbb{P}(A \cup B)$. [2]
- (b) $\mathbb{P}(A) = 0.5$, $\mathbb{P}(B) = 0.6$ and $\mathbb{P}(A \cup B) = 0.8$. Are A and B independent? [2]
- (c) For which events A is A independent of itself? [2]

9

6 marks

- (a) A coin is tossed 5 times. Find the probability that every toss shows the same face. [2]
- (b) Find the probability that the five tosses are not all the same, and include at least two heads. [2]
- (c) Independent events $E_1, \dots, E_n$ each have probability $\frac{1}{n}$. Find the probability that none of them happens, and say what happens to it as n grows. [2]

10

7 marks

Two dice are rolled. Let A be “the sum is 7”, B be “the first die shows 4” and C be “the second die shows 3”.

- (a) Find $\mathbb{P}(A)$, $\mathbb{P}(B)$ and $\mathbb{P}(C)$. [1]
- (b) Show that A , B and C are pairwise independent. [3]
- (c) Find $\mathbb{P}(A \cap B \cap C)$. [2]
- (d) Are A , B and C mutually independent? Explain in words why not. [1]

11

8 marks

Exam style. An archer hits a target with probability 0.7 on each shot, independently.

- (a) Find the probability of at least 2 hits in 3 shots. [2]
- (b) Find the probability that the first hit is on the third shot. [2]
- (c) Archers A (hit probability 0.7) and B (hit probability 0.5) shoot in turn, A first, until someone hits. Find the probability that A hits first. [3]
- (d) Are “ A hits with the first shot” and “ B hits first” independent? [1]

12

8 marks

Exam style. Two places s and t are joined by two routes: route 1 uses links a and b one after the other, and route 2 uses link c . Each link works independently with probability p .

- (a) Find the probability that route 1 works. [1]
- (b) Find the probability that s and t are connected. [2]
- (c) Evaluate this when $p = 0.9$. [1]
- (d) With $p = 0.9$, given that s and t are connected, find the probability that link c works. [2]
- (e) Are “route 1 works” and “route 2 works” independent? Are “route 1 works” and “ s and t are connected”? [2]

End of questions. Worked answers and the mark scheme are in the matching answers PDF.