

Topic 3 · Sheet A: Questions

The law of total probability and Bayes' theorem

Suggested time

About 75 minutes

Total marks

73

Questions

12

Builds on

Topics 1 to 2

How to use this sheet. Work on your own paper. The questions start gently and get harder; questions 11 and 12 are exam style. Marks are a guide to how much working is expected, roughly one minute per mark. You need partitions, the law of total probability $\mathbb{P}(B) = \sum_k \mathbb{P}(B | A_k)\mathbb{P}(A_k)$, and Bayes' theorem $\mathbb{P}(A_j | B) = \frac{\mathbb{P}(B | A_j)\mathbb{P}(A_j)}{\sum_k \mathbb{P}(B | A_k)\mathbb{P}(A_k)}$.

Tip. Give exact answers as fractions where you can. A tree diagram is a good way to organise the working.

1

4 marks

Events A and B have $\mathbb{P}(B) = 0.3$, $\mathbb{P}(A | B) = 0.5$ and $\mathbb{P}(A | B^c) = 0.2$. Find:

- (a) $\mathbb{P}(A)$; [1]
- (b) $\mathbb{P}(A \cap B)$; [1]
- (c) $\mathbb{P}(B | A)$; [1]
- (d) $\mathbb{P}(B^c | A)$. [1]

2

4 marks

A die is rolled. Is each collection of events a partition? If not, say why.

- (a) $\{1, 2\}, \{3, 4, 5\}, \{6\}$ [1]
- (b) $\{1, 2, 3\}, \{3, 4\}, \{5, 6\}$ [1]
- (c) "even", "odd" [1]
- (d) $\{1\}, \{2\}, \{3, 4\}$ [1]

3

6 marks

Three machines make 50%, 30% and 20% of a factory's output. Their items are faulty with probabilities 1%, 2% and 5%.

- (a) Find the probability that a random item is faulty. [2]
- (b) Given that an item is faulty, find the probability that it came from the third machine. [2]

- (c) Given that an item is not faulty, find the probability that it came from the first machine. [2]

4

6 marks

Bag 1 holds 3 red and 2 blue balls; bag 2 holds 1 red and 4 blue. A coin is tossed to choose a bag, and one ball is drawn from it.

- (a) Find the probability that the ball is red. [2]
(b) Given that it is red, find the probability that it came from bag 1. [2]
(c) Given that it is blue, find the probability that it came from bag 1. [2]

5

6 marks

2% of a population have a condition. A test is positive for 90% of people with the condition and for 5% of people without it.

- (a) Find the probability that a random person tests positive. [2]
(b) Given a positive test, find the probability of having the condition. [2]
(c) Given a negative test, find the probability of having the condition. [2]

6

6 marks

Bag A holds 2 white and 3 black balls; bag B holds 4 white and 1 black. A ball is drawn at random from A and put into B . Then a ball is drawn at random from B .

- (a) Find the probability that the ball drawn from B is white. [3]
(b) Given that the ball drawn from B is white, find the probability that the ball moved from A was white. [3]

7

6 marks

On a given day it rains with probability 0.2, is cloudy but dry with probability 0.5, and is sunny with probability 0.3. A bus is late with probability 0.4 if it rains, 0.1 if it is cloudy and 0.05 if it is sunny.

- (a) Find the probability that the bus is late. [2]
(b) Given that the bus is late, find the probability that it rained. [2]
(c) Given that the bus is on time, find the probability that it was sunny. [2]

8

6 marks

A bag has two coins: one fair and one with heads on both sides. One coin is chosen at random and tossed.

- (a) Find the probability of a head. [1]
(b) Given a head, find the probability that the coin is double-headed. [2]
(c) The same coin is tossed again and shows another head. Now find the probability that it is double-headed. [2]
(d) Find the probability that it is double-headed given n heads in n tosses. [1]

9

6 marks

A die is rolled, and a coin is then tossed as many times as the number shown.

- (a) Find the probability that no heads appear. [3]
- (b) Given that no heads appear, find the probability that the die showed 1. [3]

10

7 marks

A bag has 3 red and 2 green balls. One ball is taken out at random and put aside without being looked at. A second ball is then taken out at random.

- (a) Use the law of total probability to find the probability that the second ball is red. [3]
- (b) Given that the second ball is red, find the probability that the first was red. [2]
- (c) Explain why your answer to (a) is the same as the probability that the first ball is red. [2]

11

8 marks

Exam style. A box has three coins: a fair coin, a coin with $\mathbb{P}(H) = 0.8$, and a coin with $\mathbb{P}(H) = 0.2$. One coin is chosen at random and tossed twice.

- (a) Find the probability of two heads. [2]
- (b) Given two heads, find the probability that the coin chosen was the one with $\mathbb{P}(H) = 0.8$. [2]
- (c) Given two heads, find the probability that a third toss of the same coin is a head. [3]
- (d) Compare your answer to (c) with the probability of a head on the first toss. [1]

12

8 marks

Exam style. 1% of a population have a condition. A test is positive for 90% of people with it and 10% of people without it. Repeated tests on the same person are independent, given whether they have the condition.

- (a) Given one positive test, find the probability of having the condition. [3]
- (b) Given two positive tests, find the probability of having the condition. [3]
- (c) Explain why the second test makes such a large difference. [2]

End of questions. Worked answers and the mark scheme are in the matching answers PDF.