

Topic 2 · Sheet A: Questions

Conditional probability and the multiplication rule

Suggested time

About 75 minutes

Total marks

73

Questions

12

Builds on

Topic 1

How to use this sheet. Work on your own paper. The questions start gently and get harder; questions 11 and 12 are exam style. Marks are a guide to how much working is expected, roughly one minute per mark. You need the conditional probability $\mathbb{P}(A | B) = \frac{\mathbb{P}(A \cap B)}{\mathbb{P}(B)}$ for $\mathbb{P}(B) > 0$, and the multiplication rule $\mathbb{P}(A_1 \cap \dots \cap A_n) = \mathbb{P}(A_1)\mathbb{P}(A_2 | A_1) \dots \mathbb{P}(A_n | A_1 \cap \dots \cap A_{n-1})$.

Notation. A^c is the complement of A . Dice are fair and six-sided; coins are fair; “at random” means every remaining choice is equally likely.

1

4 marks

- (a) A die is rolled. Let A be “even” and B be “at least 4”. Find $\mathbb{P}(A | B)$. [1]
- (b) For the same A and B , find $\mathbb{P}(B | A)$. [1]
- (c) Two dice are rolled. Find $\mathbb{P}(\text{sum is 8} | \text{first die shows 3})$. [1]
- (d) Find $\mathbb{P}(\text{first die shows 3} | \text{sum is 8})$. [1]

2

4 marks

Events A and B have $\mathbb{P}(A) = 0.6$, $\mathbb{P}(B) = 0.5$ and $\mathbb{P}(A \cap B) = 0.2$. Find:

- (a) $\mathbb{P}(A | B)$; [1]
- (b) $\mathbb{P}(B | A)$; [1]
- (c) $\mathbb{P}(A^c | B)$; [1]
- (d) $\mathbb{P}(A | B^c)$. [1]

3

6 marks

A coin is tossed three times.

- (a) Find the probability of at least two heads. [1]
- (b) Given that the first toss is a head, find the probability of at least two heads. [2]
- (c) Given that there are exactly two heads, find the probability that the first toss is a head. [2]

- (d) Given that there is at least one head, find the probability that all three are heads. [1]

4

6 marks

A bag holds 4 red and 6 green counters. Three are taken out at random, one at a time, without replacement.

- (a) Use the multiplication rule to find the probability of red, red, green, in that order. [2]
(b) Find the probability that all three are red. [2]
(c) Find the probability that exactly two are red. [2]

5

6 marks

Two cards are dealt from a shuffled deck of 52, which has 4 aces.

- (a) Find the probability that both are aces. [2]
(b) Find the probability that the second is an ace, given that the first is an ace. [1]
(c) Find the probability that the second is an ace, given that the first is not. [1]
(d) Find the probability that at least one is an ace. [2]

6

6 marks

A box has 12 fuses, 3 of them faulty. Fuses are tested one at a time, in random order, until a faulty one is found.

- (a) Find the probability that the first fuse tested is faulty. [1]
(b) Find the probability that the first faulty fuse is the second one tested. [2]
(c) Find the probability that the first faulty fuse is the third one tested. [2]
(d) Find the probability that more than three tests are needed. [1]

7

6 marks

Two dice are rolled.

- (a) Given that at least one die shows 6, find the probability that both show 6. [2]
(b) Given that the first die shows 6, find the probability that both show 6. [2]
(c) Explain why the answers to (a) and (b) are different. [2]

8

6 marks

Let B be an event with $\mathbb{P}(B) > 0$.

- (a) Show that $\mathbb{P}(A | B) + \mathbb{P}(A^c | B) = 1$ for any event A . [2]
(b) Show that if $A \subseteq B$, then $\mathbb{P}(A | B) = \frac{\mathbb{P}(A)}{\mathbb{P}(B)}$. [2]
(c) Show that if A and B cannot both happen, then $\mathbb{P}(A | B) = 0$. [2]

9

6 marks

An urn starts with 2 white balls and 1 black ball. A ball is drawn at random, then returned together with one more ball of the same colour. This is repeated. Let W_k be the event that draw k is white, and B_k that it is black.

- (a) Find $\mathbb{P}(W_1)$. [1]
- (b) Find $\mathbb{P}(W_2 \mid W_1)$. [1]
- (c) Find $\mathbb{P}(W_1 \cap W_2 \cap W_3)$. [2]
- (d) Find $\mathbb{P}(B_1 \cap B_2)$. [2]

10

7 marks

A die is rolled four times.

- (a) Find the probability that all four results are different. [2]
- (b) Given that the first two results are different, find the probability that all four are different. [2]
- (c) Find the probability that the first repeated result happens on roll 3. [3]

11

8 marks

Exam style. A group has 5 men and 4 women. Three people are chosen at random, one at a time, without replacement.

- (a) Find the probability that the choices are woman, woman, man, in that order. [2]
- (b) Find the probability that exactly two women are chosen. [2]
- (c) Given that the first two chosen are women, find the probability that the third is a woman. [1]
- (d) Given that exactly two women are chosen, find the probability that they were the first two chosen. [3]

12

8 marks

Exam style. An inspector takes 3 bulbs at random, without replacement, from a batch of 10 bulbs, of which 4 are faulty. Let M be the event that at least one faulty bulb is taken, and E the event that exactly one is.

- (a) Find the probability that no faulty bulb is taken. [2]
- (b) Find $\mathbb{P}(M)$. [1]
- (c) Find $\mathbb{P}(E)$. [2]
- (d) Find $\mathbb{P}(E \mid M)$. [2]
- (e) Given E , find the probability that the first bulb taken is faulty. [1]

End of questions. Worked answers and the mark scheme are in the matching answers PDF.