

Topic 2 · Sheet A: Worked answers and mark scheme

Conditional probability and the multiplication rule

Suggested time

About 75 minutes

Total marks

73

Questions

12

Builds on

Topic 1

How marks are awarded. Each mark has a code, as on most UK exam papers.

B A mark for a correct statement or value on its own. It does not depend on any other mark.

M A method mark, for a correct approach, even if the final answer is wrong.

A An accuracy mark, for the correct answer *after* a correct method. It cannot be awarded without the M mark before it.

E An explanation mark, for a clear reason in words. A correct reason that is worded differently scores in full.

For true or false questions, the mark is for the correct decision; where a reason is asked for, it must be the right reason.

1 First conditional probabilities

4 marks

(a) $A \cap B = \{4, 6\}$, so $\frac{2/6}{3/6} = \frac{2}{3}$. (b) $\frac{2}{3}$. (c) The second die must show 5: $\frac{1}{6}$. (d) The sum 8 comes from $(2, 6), (3, 5), (4, 4), (5, 3), (6, 2)$: $\frac{1}{5}$.

B1 each part correct

4

Common mistake Treating (c) and (d) as the same question. Conditioning on different events gives different answers: $\frac{1}{6}$ and $\frac{1}{5}$.

2 From given probabilities

4 marks

(a) $\frac{0.2}{0.5} = \frac{2}{5}$. (b) $\frac{0.2}{0.6} = \frac{1}{3}$. (c) $1 - \frac{2}{5} = \frac{3}{5}$. (d) $\frac{\mathbb{P}(A) - \mathbb{P}(A \cap B)}{1 - \mathbb{P}(B)} = \frac{0.4}{0.5} = \frac{4}{5}$.

B1 each part correct

4

Why this question Part (d) needs $\mathbb{P}(A \cap B^c) = \mathbb{P}(A) - \mathbb{P}(A \cap B)$, which splits A into the part inside B and the part outside it.

3 Three tosses

6 marks

- (a) HHT, HTH, THH, HHH : $\frac{4}{8} = \frac{1}{2}$.
- (b) Given H first, at least one more head in two tosses: $\frac{3}{4}$.
- (c) The outcomes with exactly two heads are HHT, HTH, THH ; two start with H : $\frac{2}{3}$.
- (d) $\frac{1/8}{7/8} = \frac{1}{7}$.

B1	(a)	1
M1	(b)	1
A1	(b)	1
M1	(c) restricting to the condition	1
A1	(c)	1
B1	(d)	1

Why this question With equally likely outcomes, conditioning on B just means counting inside B instead of inside the whole sample space.

4 The multiplication rule

6 marks

- (a) $\frac{4}{10} \cdot \frac{3}{9} \cdot \frac{6}{8} = \frac{1}{10}$.
- (b) $\frac{4}{10} \cdot \frac{3}{9} \cdot \frac{2}{8} = \frac{1}{30}$.
- (c) The orders RRG, RGR, GRR each have probability $\frac{4 \cdot 3 \cdot 6}{10 \cdot 9 \cdot 8} = \frac{1}{10}$, so $\frac{3}{10}$.

M1	(a)	1
A1	(a)	1
M1	(b)	1
A1	(b)	1
M1	(c) three orders	1
A1	(c)	1

Common mistake In (c), using only the order RRG . "Exactly two red" does not say which draw is green, so all three orders count.

5 Two cards

6 marks

- (a) $\frac{4}{52} \cdot \frac{3}{51} = \frac{1}{221}$.
- (b) $\frac{3}{51} = \frac{1}{17}$.

(c) $\frac{4}{51}$.

(d) $1 - \frac{48}{52} \cdot \frac{47}{51} = \frac{33}{221}$.

M1	(a)	1
A1	(a)	1
B1	(b)	1
B1	(c)	1
M1	(d) complement	1
A1	(d)	1

Why this question Each factor in the multiplication rule is a conditional probability like (b) or (c): what is left in the deck after the earlier cards.

6 Testing fuses

6 marks

(a) $\frac{3}{12} = \frac{1}{4}$.

(b) $\frac{9}{12} \cdot \frac{3}{11} = \frac{9}{44}$.

(c) $\frac{9}{12} \cdot \frac{8}{11} \cdot \frac{3}{10} = \frac{9}{55}$.

(d) The first three are all good: $\frac{9}{12} \cdot \frac{8}{11} \cdot \frac{7}{10} = \frac{21}{55}$.

B1	(a)	1
M1	(b)	1
A1	(b)	1
M1	(c)	1
A1	(c)	1
B1	(d)	1

Why this question Check: $\frac{1}{4} + \frac{9}{44} + \frac{9}{55} + \frac{21}{55} = 1$. The four cases cover every possibility exactly once.

7 Which information?

6 marks

- (a) 11 outcomes contain a 6, and only $(6, 6)$ has two: $\frac{1}{11}$.
- (b) The second die must show 6: $\frac{1}{6}$.
- (c) “At least one 6” allows 11 outcomes, only one of them a double six. “The first die is 6” allows only 6 outcomes, one of them a double six. **The conditioning events are different sizes**, so the answers differ.

M1	(a)	1
A1	(a)	1
M1	(b)	1
A1	(b)	1
E2	(c)	2

Common mistake Assuming that knowing “a 6” is the same as knowing “the first die is 6”. Saying which die matters.

8 Properties of conditional probability

6 marks

- (a) $A \cap B$ and $A^c \cap B$ are disjoint with union B , so $\mathbb{P}(A \cap B) + \mathbb{P}(A^c \cap B) = \mathbb{P}(B)$. Divide by $\mathbb{P}(B)$.
- (b) If $A \subseteq B$ then $A \cap B = A$, so $\mathbb{P}(A | B) = \frac{\mathbb{P}(A)}{\mathbb{P}(B)}$.
- (c) $A \cap B = \emptyset$, so $\mathbb{P}(A \cap B) = 0$ and $\mathbb{P}(A | B) = 0$.

M1	(a) splitting B	1
A1	(a)	1
M1	(b) $A \cap B = A$	1
A1	(b)	1
M1	(c)	1
A1	(c)	1

Why this question Conditioning on B behaves like a new probability, with B playing the role of the whole sample space.

9 An urn that changes

6 marks

- (a) $\frac{2}{3}$.
- (b) After a white draw there are 3 white and 1 black: $\frac{3}{4}$.
- (c) $\frac{2}{3} \cdot \frac{3}{4} \cdot \frac{4}{5} = \frac{2}{5}$.

(d) $\frac{1}{3} \cdot \frac{2}{4} = \frac{1}{6}$.

B1	(a)	1
B1	(b)	1
M1	(c)	1
A1	(c)	1
M1	(d)	1
A1	(d)	1

Common mistake Using $\frac{2}{3}$ for every draw. The urn changes after each draw, so each factor must be worked out from the urn at that moment.

10 Repeated results

7 marks

(a) $\frac{6}{6} \cdot \frac{5}{6} \cdot \frac{4}{6} \cdot \frac{3}{6} = \frac{5}{18}$.

(b) Rolls 3 and 4 must avoid the earlier values: $\frac{4}{6} \cdot \frac{3}{6} = \frac{1}{3}$.

(c) Roll 2 differs from roll 1, then roll 3 matches one of them: $\frac{5}{6} \cdot \frac{2}{6} = \frac{5}{18}$.

M1	(a)	1
A1	(a)	1
M1	(b)	1
A1	(b)	1
M1	(c) roll 2 differs	1
M1	(c) roll 3 matches	1
A1	(c)	1

Why this question Check (b) with the definition: $\frac{5/18}{5/6} = \frac{1}{3}$. The multiplication rule and the definition always agree.

11 Exam style: choosing a committee

8 marks

- (a) $\frac{4}{9} \cdot \frac{3}{8} \cdot \frac{5}{7} = \frac{5}{42}$.
- (b) The orders WWM , WMW , MWW each have probability $\frac{5}{42} \cdot \frac{15}{42} = \frac{5}{14}$.
- (c) 2 women are left among 7 people: $\frac{2}{7}$.
- (d) $\frac{\mathbb{P}(WWM)}{\mathbb{P}(\text{exactly two women})} = \frac{5/42}{15/42} = \frac{1}{3}$.

M1	(a)	1
A1	(a)	1
M1	(b) three orders	1
A1	(b)	1
B1	(c)	1
M1	(d) intersection is WWM	1
M1	(d) definition	1
A1	(d)	1

Why this question Part (d) can also be seen by symmetry: given exactly two women, the man is equally likely to be in any of the three positions.

12 Exam style: faulty bulbs

8 marks

- (a) $\frac{6}{10} \cdot \frac{5}{9} \cdot \frac{4}{8} = \frac{1}{6}$.
- (b) $1 - \frac{1}{6} = \frac{5}{6}$.
- (c) Three positions for the faulty bulb, each $\frac{4}{10} \cdot \frac{6}{9} \cdot \frac{5}{8} = \frac{1}{6} \cdot \frac{1}{2}$.
- (d) $E \subseteq M$, so $\mathbb{P}(E | M) = \frac{\mathbb{P}(E)}{\mathbb{P}(M)} = \frac{1/2}{5/6} = \frac{3}{5}$.
- (e) By symmetry the faulty bulb is equally likely to be first, second or third: $\frac{1}{3}$.

M1	(a)	1
A1	(a)	1
B1	(b)	1
M1	(c)	1
A1	(c)	1
M1	(d) $E \subseteq M$	1
A1	(d)	1
B1	(e)	1

Common mistake In (d), multiplying $\mathbb{P}(E)\mathbb{P}(M)$ for the intersection. Here $E \cap M = E$, because exactly one faulty bulb already means at least one.

Total: 73 marks. Spotted a mistake? Report it at unimixedtutor.com and we will fix it.