

Topic 1 · Sheet A: Questions

Probability and problem solving

Suggested time

About 75 minutes

Total marks

73

Questions

12

Builds on

Nothing earlier

How to use this sheet. Work on your own paper. The questions start gently and get harder; questions 11 and 12 are exam style. Marks are a guide to how much working is expected, roughly one minute per mark. You need probability from school: equally likely outcomes, complements, adding probabilities of outcomes that cannot happen together, multiplying for independent tosses or rolls, geometric series, and the trick of ignoring outcomes that do not affect the event.

Notation. $\mathbb{P}(A)$ is the probability of A . Dice are fair, six-sided and numbered 1 to 6; coins are fair unless stated; repeated tosses and rolls are independent. A biased coin shows heads with probability p and tails with $q = 1 - p$.

1

4 marks

Two dice are rolled. Find the probability that:

- (a) the sum is 7; [1]
- (b) both dice show the same number; [1]
- (c) at least one die shows a 6; [1]
- (d) the sum is at least 10. [1]

2

4 marks

A coin is tossed four times. Find the probability of:

- (a) exactly two heads; [1]
- (b) at least one head; [1]
- (c) the first head appearing on the third toss; [1]
- (d) the sequence H, H, T, T in that order. [1]

3

6 marks

A bag holds 5 red and 3 blue balls. Two are taken out at random, one after the other, without replacement.

- (a) Find the probability that both are red. [2]
- (b) Find the probability that one is red and one is blue. [2]
- (c) Find the probability that the second ball is red. [2]

4

6 marks

A die is rolled until the first 6 appears.

- (a) Find the probability that the first 6 is on roll n . [2]
- (b) Find the probability that a 6 appears within the first three rolls. [2]
- (c) Find the probability that the first 6 is on an even-numbered roll. [2]

5

6 marks

A die is rolled repeatedly. For each part, decide which outcomes can be ignored, and use this to find the probability.

- (a) A 1 appears before the first 2. [2]
- (b) A 5 or a 6 appears before the first 1. [2]
- (c) Exactly one 5 appears before the first 1. [2]

6

6 marks

Person 1 tosses a coin once and Person 2 tosses a coin twice.

- (a) Find the probability that Person 2 gets more heads than Person 1. [3]
- (b) Find the probability that they get the same number of heads. [2]
- (c) Hence find the probability that Person 1 gets more heads. [1]

7

6 marks

A biased coin is tossed repeatedly.

- (a) Find the probability that the first two tosses are H then T . [1]
- (b) Find the probability that the first two tosses are different. [1]
- (c) Let N be the toss on which the first head appears. Find $\mathbb{P}(N = n)$. [2]
- (d) Find $\mathbb{P}(N > n)$. [2]

8

6 marks

A and B take turns to roll a die, A first. The first to roll a 6 wins.

- (a) Find the probability that A wins. [3]
- (b) Find the probability that B wins. [1]
- (c) Find the probability that nobody has won after 4 rolls. [2]

9

6 marks

Three dice are rolled. Find the probability that:

- (a) all three numbers are different; [2]
- (b) the sum is 4; [2]

- (c) the largest number is 6. [2]

10

7 marks

A biased coin is tossed repeatedly. Let N be the first toss whose face differs from the face on toss 1.

- (a) Find $\mathbb{P}(N = 2)$. [2]
(b) Find $\mathbb{P}(N = 3)$, and simplify it. [2]
(c) Find $\mathbb{P}(N = n)$ for $n \geq 2$. [2]
(d) Simplify your answer to (c) for a fair coin. [1]

11

8 marks

Exam style. A coin is tossed until two tosses in a row show the same face. Let N be the number of tosses made.

- (a) Find $\mathbb{P}(N = 2)$. [1]
(b) Explain why $\mathbb{P}(N = n) = \left(\frac{1}{2}\right)^{n-1}$ for $n \geq 2$. [3]
(c) Check that these probabilities add up to 1. [2]
(d) Find the probability that N is even. [2]

12

8 marks

Exam style. A player rolls two dice. A sum of 7 or 11 wins at once; a sum of 2, 3 or 12 loses at once; any other sum means the player rolls both dice again, and the game repeats.

- (a) Find the probability of winning on the first roll. [2]
(b) Find the probability of losing on the first roll. [1]
(c) Find the probability that the game continues after the first roll. [1]
(d) Find the probability that the player eventually wins. [3]
(e) Find the probability that the game ends on exactly the third roll. [1]

End of questions. Worked answers and the mark scheme are in the matching answers PDF.