

Topic 1 · Sheet A: Questions

Inequalities: the order on $\mathbb{R}$, graphs and case analysis

Suggested time

About 75 minutes

Total marks

73

Questions

12

Builds on

Nothing earlier

How to use this sheet. Work on your own paper. The questions start gently and get harder; questions 11 and 12 are exam style. Marks are a guide to how much working is expected, roughly one minute per mark. You need the order properties of $\mathbb{R}$ (compatibility with addition and with positive multiplication, transitivity, trichotomy), interval notation, solving inequalities with graphs, and case analysis with sign tables. Absolute values come in Topic 2.

Notation. In this module, (a, b) , $[a, b]$, $(a, b]$, $[a, b)$, (a, ∞) and so on are intervals, and $\mathbb{R}^+ = \{x \in \mathbb{R} : x > 0\}$. Give every solution set as an interval or a union of intervals. All variables are real.

1

4 marks

Decide whether each inequality is always true, never true, or true for some values of the variables only.

(a) $x^2 + 4 \geq 0$ [1]

(b) $x + 1 < x$ [1]

(c) $x^2 > 4$ [1]

(d) $(x - 1)^2 + (y + 2)^2 < 0$ [1]

2

4 marks

Write each set as an interval or a union of intervals.

(a) $\{x \in \mathbb{R} : -2 < x \leq 5\}$ [1]

(b) $\{x \in \mathbb{R} : x \geq 3\}$ [1]

(c) $\{x \in \mathbb{R} : x < 1 \text{ or } x > 4\}$ [1]

(d) $\{x \in \mathbb{R} : 0 \leq x < 1\} \cap \{x \in \mathbb{R} : x > \frac{1}{2}\}$ [1]

3

6 marks

Solve each inequality, saying which order property you use at each step.

(a) $3x - 5 < 7$ [1]

(b) $2 - 3x \geq 8$ [2]

(c) $-4 < 2x + 2 \leq 10$ [2]

(d) $x - 1 \leq x + 5$ [1]

4

6 marks

Use graphs to find all $x \in \mathbb{R}$ satisfying each inequality. In each case, say where the two graphs meet.

(a) $x^2 < 2x$ [2]

(b) $x^3 \geq x$ [2]

(c) $\frac{1}{x} > 1$ [2]

5

6 marks

Use case analysis to solve each inequality.

(a) $(x - 2)(x + 3) > 0$ [2]

(b) $(x - 2)(x + 3) \leq 0$ [2]

(c) $x^2 - 5x + 4 < 0$ [2]

6

6 marks

Use a sign table to solve each inequality.

(a) $(x - 1)(x + 2)(x - 4) > 0$ [3]

(b) $x(x - 3)^2 \geq 0$ [3]

7

6 marks

Solve each inequality. Take care with the points where a denominator is zero.

(a) $\frac{x + 1}{x - 2} < 0$ [2]

(b) $\frac{x - 3}{x + 1} \geq 0$ [2]

(c) $\frac{x}{x - 1} > 0$ [2]

8

6 marks

For each statement about real numbers a and b , decide whether it is true. If it is false, give values of a and b that show this. If it is true, name the order properties that give it.

(a) If $a < b$ then $a^2 < b^2$. [2]

(b) If $0 < a < b$ then $a^2 < b^2$. [2]

(c) If $a < b$ then $-2a > -2b$. [2]

9

7 marks

Write each side as a single fraction before solving. Do not multiply through by the denominators.

(a) Find all $x \in \mathbb{R}$ with $\frac{1}{x+1} < \frac{1}{x+4}$. [3]

(b) Find all $x \in \mathbb{R}$ with $\frac{2}{x-1} \geq \frac{1}{x+1}$. [4]

10

6 marks

Let $g(x) = \frac{(x-1)(x-2)}{(x+3)(x-4)}$.

(a) Check that $x = 0$ satisfies $g(x) < 0$. [1]

(b) Use a sign table to find every x with $g(x) < 0$. [4]

(c) Write down an inequality in one variable that has no solutions. [1]

11

8 marks

Exam style.

(a) Solve $x^3 - x^2 - 6x > 0$. [3]

(b) Solve $\frac{x^2 - 1}{x^2 - 4} \leq 0$. [4]

(c) The step $x + 2 > 0 \Rightarrow x + 3 > 0$ is valid, but it is not an equivalence. Give a value of x that shows this. [1]

12

8 marks

Exam style.

(a) Find all $x \in \mathbb{R}$ with $x^2 < x < x^3$. Solve each of the two inequalities separately, then combine them. [4]

(b) Find all $x \in \mathbb{R}$ with $\frac{1}{x} < x$. [3]

(c) Which x satisfy both part (b) and $x < 2$? [1]

End of questions. Worked answers and the mark scheme are in the matching answers PDF.