

Topic 5 · Sheet A: Questions

Union, intersection, difference and the boolean algebra of sets

Suggested time

About 75 minutes

Total marks

74

Questions

12

Builds on

Topics 1 to 4

How to use this sheet. Work on your own paper. The questions start gently and get harder; questions 11 and 12 are exam style. Marks are a guide to how much working is expected, roughly one minute per mark. You need unions, intersections, set difference and the laws relating them, plus Topics 1 to 4.

Notation. $X \cup Y$, $X \cap Y$ and $X - Y = \{x \in X \mid x \notin Y\}$ are the union, intersection and difference. For a set $\mathcal{X}$ whose elements are sets, $\bigcup_{X \in \mathcal{X}} X$ and $\bigcap_{X \in \mathcal{X}} X$ are the union and intersection over $\mathcal{X}$. Sets with empty intersection are disjoint. "Or" always means at least one.

1

4 marks

Let $X = \{0, 1, 2, 3\}$ and $Y = \{2, 3, 4\}$. Find each set.

- (a) $X \cup Y$ [1]
- (b) $X \cap Y$ [1]
- (c) $X - Y$ [1]
- (d) $Y - X$ [1]

2

4 marks

Write each set as $\llbracket n \rrbracket$ for some n , or as a list.

- (a) $\llbracket 3 \rrbracket \cup \llbracket 5 \rrbracket$ [1]
- (b) $\llbracket 3 \rrbracket \cap \llbracket 5 \rrbracket$ [1]
- (c) $\llbracket 5 \rrbracket - \llbracket 3 \rrbracket$ [1]
- (d) $\llbracket 3 \rrbracket - \llbracket 5 \rrbracket$ [1]

3

5 marks

Let $X = \{a, b\}$. Find each set.

- (a) $X \cup \emptyset$ [1]
- (b) $X \cap \emptyset$ [1]

- (c) $X - \emptyset$ [1]
 (d) $\emptyset - X$ [1]
 (e) $X - X$ [1]

4

6 marks

Let $A = \{n \in \mathbb{Z}_{20} \mid n \text{ is even}\}$, $B = \{n \in \mathbb{Z}_{20} \mid n \text{ is a multiple of 3}\}$ and $C = \{n \in \mathbb{Z}_{20} \mid n < 6\}$.

- (a) List $A \cap B$. [1]
 (b) List $B - A$. [1]
 (c) List $(A \cap B) \cup C$. [1]
 (d) List $C - (A \cup B)$. [1]
 (e) Find $|A|$, $|B|$ and $|A \cup B|$, and check that $|A \cup B| = |A| + |B| - |A \cap B|$. [2]

5

6 marks

Find the union and the intersection over each set $\mathcal{X}$.

- (a) $\mathcal{X} = \{\llbracket 1 \rrbracket, \llbracket 3 \rrbracket, \llbracket 4 \rrbracket\}$ [2]
 (b) $\mathcal{X} = \{\{0, 1\}, \{1, 2\}, \{2, 3\}\}$ [2]
 (c) $\mathcal{X} = \{\llbracket n \rrbracket \mid n \in \mathbb{N}\}$ [2]

6

5 marks

- (a) Are $\llbracket 3 \rrbracket$ and $\{3, 4\}$ disjoint? [1]
 (b) Are $\{n \in \mathbb{N} \mid n \text{ is even}\}$ and $\{n \in \mathbb{N} \mid n \text{ is a multiple of 3}\}$ disjoint? [1]
 (c) Suppose that $|X| = 7$, $|Y| = 5$ and $|X \cap Y| = 2$. Find $|X \cup Y|$. [1]
 (d) Suppose that $X \cup Y = X$. What must be true of X and Y ? [1]
 (e) Suppose that $X \cap Y = X$. What must be true of X and Y ? [1]

7

6 marks

Let $X = \llbracket 6 \rrbracket$, $Y = \{0, 2, 4, 6, 8\}$ and $Z = \{1, 2, 3, 8, 9\}$.

- (a) List $X \cap Y \cap Z$. [1]
 (b) List $(X \cup Y) - Z$. [1]
 (c) List $X - (Y \cup Z)$ and $(X - Y) \cap (X - Z)$. Which law do they illustrate? [2]
 (d) List $X - (Y \cap Z)$ and $(X - Y) \cup (X - Z)$. [2]

8

6 marks

Let $X = \{0, 1, 2\}$, $Y = \{1, 2, 3\}$ and $Z = \{2, 3, 4\}$.

- (a) Find $X \cap (Y \cup Z)$ and $(X \cap Y) \cup (X \cap Z)$. [2]
- (b) Find $X \cup (Y \cap Z)$ and $(X \cup Y) \cap (X \cup Z)$. [2]
- (c) Find $(X - Y) - Z$ and $X - (Y - Z)$. What does this show about set difference? [2]

9

7 marks

Let $A = \llbracket 2 \rrbracket$ and $B = \{1, 2\}$.

- (a) List $\mathcal{P}(A) \cap \mathcal{P}(B)$. Which power set is it? [2]
- (b) List $\mathcal{P}(A) \cup \mathcal{P}(B)$. [2]
- (c) Is $\mathcal{P}(A) \cup \mathcal{P}(B) = \mathcal{P}(A \cup B)$? Explain, using a specific subset. [3]

10

6 marks

- (a) How many subsets S of $\llbracket 6 \rrbracket$ satisfy $S \cap \{0, 1\} = \emptyset$? [1]
- (b) How many subsets S of $\llbracket 6 \rrbracket$ satisfy $S \cup \{0, 1\} = \llbracket 6 \rrbracket$? [2]
- (c) How many ordered pairs (S, T) of subsets of $\llbracket 3 \rrbracket$ have $S \cap T = \emptyset$? [3]

11

10 marks

Exam style. Let $U = \llbracket 12 \rrbracket$, and let $A = \{n \in U \mid n \text{ is even}\}$, $B = \{n \in U \mid n \geq 7\}$ and $C = \{2, 3, 5, 7, 11\}$.

- (a) List $A \cap B$ and $A \cup B$. [2]
- (b) List $U - (A \cup B)$ and $(U - A) \cap (U - B)$, and explain why they agree. [3]
- (c) List $A \cup B \cup C$, and find its cardinality. [2]
- (d) List $(A - B) \cup (B - A)$, and check that it equals $(A \cup B) - (A \cap B)$. [3]

12

9 marks

Exam style. Each expression below equals one of X , Y , $X \cap Y$, $X \cup Y$ or $\emptyset$, for every pair of sets X and Y . Decide which, then check your answer with $X = \{0, 1, 2\}$ and $Y = \{1, 2, 3\}$.

- (a) $X \cap (X \cup Y)$ [2]
- (b) $X \cup (X \cap Y)$ [1]
- (c) $(X - Y) \cup (X \cap Y)$ [2]
- (d) $X - (X - Y)$, giving a reason that works for every X and Y [3]
- (e) $(X - Y) \cap (Y - X)$ [1]

End of questions. Worked answers and the mark scheme are in the matching answers PDF.