

Topic 14 · Sheet A: Questions

Euler circuits and Euler walks

Suggested time
About 75 minutes

Total marks
73

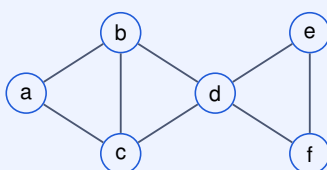
Questions
12

Builds on
Topics 1 to 13

How to use this sheet. Work on your own paper. The questions start gently and get harder; questions 11 and 12 are exam style. Marks are a guide to how much working is expected, roughly one minute per mark. You need walks and closed walks, Euler circuits (closed walks using every edge exactly once), Euler walks (walks using every edge exactly once that are not closed), and the two results: a connected graph has an Euler circuit exactly when every degree is even, and an Euler walk exactly when exactly two degrees are odd.

Notation. Write a walk as its list of nodes, such as a, b, c, a . In a multigraph, a loop adds 2 to the degree.

The graph G .



1

4 marks

For each connected graph, say whether it has an Euler circuit, an Euler walk, or neither.

- (a) C_5 [1]
- (b) P_4 [1]
- (c) The complete bipartite graph with parts of sizes 2 and 4 [1]
- (d) A multigraph with four nodes of degrees 3, 3, 3, 5 [1]

2

4 marks

These parts are about G .

- (a) Which nodes have odd degree? [1]
- (b) Does G have an Euler circuit? [1]
- (c) Does G have an Euler walk? If so, where must it start and end? [1]
- (d) Write down an Euler walk in G . [1]

3

6 marks

The house graph has a square 1, 2, 3, 4 (edges 12, 23, 34, 41) and a roof node 5 joined to 1 and 2.

- (a) Write down the degree of each node. [2]
- (b) Decide whether the house has an Euler circuit or an Euler walk. [2]
- (c) Find an Euler walk or circuit, whichever exists. [2]

4

6 marks

The bowtie has two triangles 1, 2, 3 and 3, 4, 5 that share node 3.

- (a) Starting at 1, a closed walk 1, 2, 3, 1 is found. Which edges does it miss? [1]
- (b) The missing edges form a graph with every degree even. Find an Euler circuit of it. [2]
- (c) Splice the two closed walks together into an Euler circuit of the bowtie. [2]
- (d) Check that your circuit has the right number of edges. [1]

5

6 marks

Add a second edge between b and c to G , giving a multigraph G' .

- (a) Explain why G' has an Euler circuit. [2]
- (b) Find an Euler circuit of G' that uses the new edge last. [2]
- (c) Remove the new edge from your circuit. What do you get in G ? [2]

6

6 marks

The 2×3 grid has top row p, q, r , bottom row s, t, u , and edges $pq, qr, st, tu, ps, qt, ru$.

- (a) Which nodes have odd degree? What does this tell you? [2]
- (b) Find an Euler walk. [2]
- (c) Which single extra edge (possibly a repeated one) would give a graph with an Euler circuit? [2]

7

6 marks

For each graph, say whether it has an Euler circuit, an Euler walk, or neither, with a reason.

- (a) The cube graph Q_3 (every node has degree 3) [2]
- (b) The complete bipartite graph with parts of sizes 3 and 3 [1]
- (c) The complete bipartite graph with parts of sizes 2 and 3 [2]
- (d) The complete bipartite graph with parts of sizes 4 and 4 [1]

8

6 marks

The envelope has the square 1, 2, 3, 4 (edges 12, 23, 34, 41), both diagonals 13 and 24, and a flap node 5 joined to 1 and 2. You want to draw it without lifting your pen or going over a line twice.

- (a) Write down the degree of each node. [2]
- (b) Where must you start and finish? [2]
- (c) Give an order in which to draw the lines. [2]

9

6 marks

A town has two banks N and S and two islands A and B . There are 2 bridges between A and N , 2 between A and S , and one bridge each between A and B , B and N , and B and S .

- (a) Model the town as a multigraph and give each degree. [2]
- (b) Can you walk crossing every bridge exactly once? [1]
- (c) One new bridge is built from B to N . Now where can such a walk start and end? [2]
- (d) Add one more bridge so that a closed walk crossing every bridge exactly once is possible. [1]

10

7 marks

A graph is the cycle 1, 2, 3, 4, 5, 6 together with the triangle 1, 3, 5 (edges 13, 35, 51).

- (a) Show that every node has even degree. [1]
- (b) Starting at 1, write down the closed walk around the triangle. [1]
- (c) The edges it misses form the 6-cycle. Write down a closed walk around it starting at 1. [2]
- (d) Splice the two walks into an Euler circuit. [2]
- (e) How many edges does the Euler circuit have? [1]

11

8 marks

Exam style.

- (a) Explain why, if a graph has an Euler circuit, every node has even degree. [2]
- (b) A connected graph has exactly two nodes of odd degree, v and w . Explain how the Euler circuit result gives an Euler walk from v to w . [3]
- (c) The diamond has nodes 1, 2, 3, 4 and edges 12, 13, 23, 24, 34. Find its odd nodes and an Euler walk. [3]

12

8 marks

Exam style. A multigraph has nodes A, B, C , two edges between A and B , two edges between B and C , one edge between C and A , and a loop at C .

- (a) Write down the degree of each node. [2]
- (b) Explain why there is an Euler walk, and say where it starts and ends. [2]

- (c) Write down an Euler walk, naming which copy of each repeated edge you use. [3]
- (d) Would one more edge between A and C give an Euler circuit? Explain. [1]

End of questions. Worked answers and the mark scheme are in the matching answers PDF.