

Topic 8 · Sheet A: Questions

Introduction to graphs: special graphs, subgraphs and bipartite graphs

Suggested time

About 75 minutes

Total marks

73

Questions

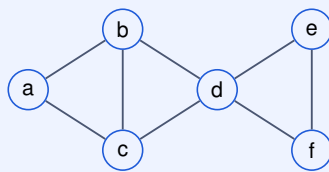
12

Builds on

Topics 1 to 7

How to use this sheet. Work on your own paper. The questions start gently and get harder; questions 11 and 12 are exam style. Marks are a guide to how much working is expected, roughly one minute per mark. You need the definitions of a graph, adjacent and incident, neighbourhoods $N(v)$ and $N[v]$, multigraphs and self-loops, P_n , C_n and K_n , bipartite graphs, subgraphs and induced subgraphs, and paths, cycles and cliques in a graph.

The graph G . Several questions use this graph G .



1

4 marks

These parts are about the graph G drawn above.

- (a) Write down $|V|$ and $|E|$. [1]
- (b) Write down $N(d)$. [1]
- (c) Write down $N[a]$. [1]
- (d) Are a and d adjacent? [1]

2

4 marks

How many edges does each graph have?

- (a) K_5 [1]
- (b) C_7 [1]
- (c) P_6 [1]
- (d) K_n [1]

3

6 marks

A graph has nodes u, v, w, x and five edges, with these endpoints.

Edge	e_1	e_2	e_3	e_4	e_5
Endpoints	(u, v)	(u, v)	(v, w)	(w, w)	(w, x)

- (a) Is it a multigraph? Explain. [1]
- (b) Which edge is a self-loop? [1]
- (c) Draw the graph. [2]
- (d) Which edges must be removed to leave a graph with no repeated edges and no loops? Which special graph is left? [2]

4

6 marks

These parts are about G .

- (a) Let H_1 have nodes $\{a, b, c\}$ and edges $\{\{a, b\}, \{a, c\}\}$. Is H_1 a subgraph of G ? Is it an induced subgraph? [2]
- (b) Write down the edges of the subgraph of G induced by $\{b, c, d, e\}$. [2]
- (c) How many subgraphs of G have node set exactly $\{a, b, c\}$? How many of them are induced? [2]

5

6 marks

These parts are about G .

- (a) Give a path of length 3 from a to f . Is there a path of length 2 from a to f ? [2]
- (b) Give a cycle of length 4 in G , listing its edges. [2]
- (c) What is the largest size of a clique in G ? List every clique of that size. [2]

6

6 marks

- (a) Show that C_4 is bipartite by giving the two parts. [1]
- (b) Explain why K_3 is not bipartite. [2]
- (c) The graph H is the cycle C_6 on nodes $1, 2, 3, 4, 5, 6$ (in that order) with the extra edge $\{1, 4\}$. Show that H is bipartite. [2]
- (d) Would H still be bipartite if the extra edge were $\{1, 3\}$ instead? [1]

7

6 marks

- (a) What is the largest number of edges in a graph with n nodes and no multi-edges or loops? [1]
- (b) How many such graphs are there on the node set $\{1, 2, 3\}$? On $\{1, \dots, n\}$? [2]
- (c) A bipartite graph has $|L| = 3$ and $|R| = 4$. What is the largest number of edges it can have? [1]

(d) What is the largest number of edges in a bipartite graph with 10 nodes? [2]

8

6 marks

Count these subgraphs of K_4 .

(a) How many cliques of size 3 (triangles) does K_4 contain? How many does K_n contain? [2]

(b) How many paths of length 2 does K_4 contain? [2]

(c) How many cycles of length 4 does K_4 contain? [2]

9

6 marks

Let C_5 have nodes 1, 2, 3, 4, 5 in order around the cycle.

(a) Which graph is the subgraph induced by $\{1, 2, 3\}$? [1]

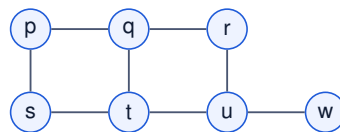
(b) Which graph is the subgraph induced by $\{1, 3\}$? [1]

(c) How many 3-node sets induce a path P_3 ? What do the other 3-node sets induce? [3]

(d) Does C_5 contain a clique of size 3? [1]

10

7 marks



(a) Show that this graph is bipartite by giving the parts L and R . [3]

(b) Is it still bipartite after adding the edge $\{p, t\}$? Justify your answer. [2]

(c) Is it still bipartite after adding the edge $\{p, u\}$ instead? Justify your answer. [2]

11

8 marks

Exam style. A graph has nodes 1, 2, 3, 4, 5, 6, and $\{i, j\}$ is an edge exactly when $|i - j|$ is 1 or 3.

(a) List the edges and give $|E|$. [2]

(b) Write down $N(2)$ and $N[4]$. [2]

(c) Show that the graph is bipartite. [2]

(d) Find a cycle of length 4. Explain why the graph has no cycle of length 3. [2]

12

8 marks

Exam style. Five people A, B, C, D, E use a social network. A is friends with B, C and D ; B is friends with C ; D is friends with E . There are no other friendships.

(a) Write the friendship graph as (V, E) . [1]

(b) Find the largest clique. [2]

(c) Find a path from C to E , and give its length. [1]

- (d) Which graph is the subgraph induced by $\{A, C, D, E\}$? [2]
- (e) What is the smallest number of new friendships needed for the graph to contain a clique of size 4? [2]

End of questions. Worked answers and the mark scheme are in the matching answers PDF.